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Thickness of the diffusive sublayer in turbulent convection

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In this paper we address the following question: how thick is the diffusive fraction of the thermal boundary layer in highly turbulent thermal convection? We have studied this problem in a large-scale Rayleigh-Bénard experiment at fixed aspect ratio $\Gamma=1.13$ and variable Rayleigh number $5.2\times10^{10}<\mathrm{Ra}<9.6\times10^{11}$ using air as the working fluid. By measuring profiles of the mean temperature in the vicinity of the heated bottom plate with a higher spatial resolution than in any other previous experiment and by complementing them with simultaneous independent measurements of the local heat flux, we have determined the shares of diffusive and convective vertical heat fluxes inside the thermal boundary layer. Our measurements show that the thickness of the sublayer where heat is exclusively transported by diffusion is only about 1/25 of the nominal thickness of the thermal boundary layer and depends only weakly on Ra in the parameter domain investigated here. This result implies that phenomenological theories which assume a diffusive heat transport in the whole thermal boundary layer are unlikely to be correct at very high Rayleigh numbers.

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I. INTRODUCTION

The heat transfer from a hot or cold solid surface to a surrounding fluid is a basic phenomenon in many natural processes or technical applications. Its prediction requires the description of the thin layer very close to the wall—the diffusive sublayer—where heat is transported exclusively by diffusion. We report highly resolved temperature measurements in turbulent Rayleigh-Bénard (RB) convection at $\mathrm{Pr}=0.7$ complemented by a local heat flux measurement at the solid surface which demonstrate that this layer spans a much smaller fraction of the full thermal boundary layer than usually assumed.

It is almost 80 years ago that Prandtl has studied for the first time the convective heat transfer in a systematic manner and developed the concept of the thermal boundary layer [1]. The main message of this early work is that this process is controlled by a very thin layer close to the solid surface. This layer in which the temperature drops or rises from the wall temperature to the temperature of the surrounding fluid is called the convective boundary layer. Its extent is usually much smaller than the size of the entire flow domain. In the close vicinity of the wall there is an even thinner layer called the diffusive sublayer (see, e.g., [2]). In this region heat is exclusively conducted by diffusion while in the outer part of the boundary layer advection plays the predominant role in heat transport. This view is widely accepted today. However, a precise prediction of the convective heat transfer in typical applications such as throughout the atmospheric boundary layer (see, e.g., [3,4]), in high power heat exchangers (see, e.g., [5]) or in naturally ventilated buildings (see, e.g., [6]) requires a deeper understanding of the structure of the convective boundary layer in general and of the diffusive sublayer in particular. Our work here aims at answering the following question: how thick is the diffusive sublayer compared with the entire extent of the convective boundary layer? We studied this problem in turbulent RB convection where a fluid layer is heated from below and cooled from above. In this model experiment the heat transport throughout the fluid is usually expressed in terms of the dimensionless Nusselt number $\mathrm{Nu}=Q/Q_d$ and it is still a challenge today to predict its dependence on the input parameters Rayleigh number $\mathrm{Ra}=(\beta g\Delta T H^3)/(\nu\kappa)$, aspect ratio $\Gamma=D/H$, and Prandtl number $\mathrm{Pr}=\nu/\kappa$ particularly in case of very high Rayleigh numbers $\mathrm{Ra}>10^9$. In the above definitions Q and Q_d stand for the total and the diffusive heat flux, β is the isobaric thermal expansion coefficient, g is the acceleration of gravity, ΔT is the temperature difference between both horizontal plates, D and H are the lateral and the vertical dimension of the fluid layer, and ν and κ are the kinematic viscosity and the thermal diffusivity of the fluid. Almost all phenomenological theories predicting $\mathrm{Nu}(\mathrm{Ra},\Gamma,\mathrm{Pr})$ are based on assumptions about the heat transport mechanism throughout the boundary layer at the heated bottom and the cooled top plate [7–9]. For Rayleigh numbers below a critical value $\mathrm{Ra_c}$ a laminar boundary layer of Blasius type (see [10,11]) is the preferred model while above this limit the boundary layer is expected to become turbulent [12]. Measurements in high Rayleigh number RB convection in water (up to $\mathrm{Ra}=10^{12}$) undertaken by Belmonte *et al.* (1993) [13], Lui and Xia (1998) [14] or Shang *et al.* (2004) [15], and direct numerical simulations by Shiskina and Thess (2009) [16] seem to confirm the hypothesis of the laminar boundary layer where diffusion is the predominant heat transport process. However, recent highly resolved measurements at large-scale experimental facilities with air as working fluid contradict them and exhibit a power-law dependence $T\sim z^{\alpha}$ [17,18]. Certainly it is undisputed that holding the argument of the nonslip boundary condition $v|_{z=0}=0$ a diffusive sublayer must exist very close to the wall [10,19,20]. The question which we wish to address here is: how thick is the diffusive sublayer in highly turbulent RB convection really?

II. EXPERIMENTAL FACILITY AND MEASUREMENT SETUP

The measurements were undertaken in a large-scale Rayleigh-Bénard experiment called the "Barrel of Ilmenau."

TABLE I. Set of parameters and summary of results of the temperature and the local heat flux measurements at constant aspect ratio $\Gamma=1.13$. The global Nusselt numbers Nu_g are derived from previous heat flux measurements and computed using a fit $Nu_g=0.0573Ra^{0.336}$.

ΔT (K)	Ra (10^{11})	q_w (W/m^2)	$dT/dz\|_{z=0}$ (K/mm)	Nu_l	Nu_g	δ_{th} (mm)	δ_q (mm)	δ_d (mm)
2.6	0.52	9.3	0.35	901	230	3.93	0.83	0.074
4.0	0.87	14.6	0.55	924	273	3.47	0.85	0.069
7.4	1.60	27.7	1.04	986	336	3.02	0.76	0.068
10.0	2.16	39.2	1.46	1020	371	2.92	0.62	0.088
15.0	3.18	59.4	2.20	1120	423	2.81	0.68	0.100
20.0	4.27	85.7	3.15	1130	467	2.69	0.76	0.098
25.0	5.27	109	3.99	1210	501	2.61	0.82	0.101
30.0	6.31	137	4.98	1240	533	2.54	0.80	0.113
35.0	7.32	162	5.83	1260	560	2.49	0.85	0.111
40.0	8.32	190	6.80	1300	585	2.42	0.78	0.109
60.0	9.59	313	10.5	1350	613	2.33	0.83	0.106

In this facility, an adiabatic cylinder with 7.15 m in diameter and 6.30 m in height in which air ($Pr=0.7$) is heated from below and cooled from above turbulent convection can be studied in unrivalled detail up to Rayleigh numbers of $Ra=10^{12}$. Because of the large size it permits temperature and velocity measurements particularly inside the boundary layer which are better resolved than in any other actual RB experiment working at comparable high Ra numbers. A detailed description of the experimental facility and the measurement technique can be found in [17]. In order to increase accuracy of the boundary conditions the electrical heating plate has been covered by an aluminum overlay in which water circulates. It homogenizes the local surface temperature to deviate less than 1 K from the mean. We have measured wall-normal temperature profiles $T(z)$ in the vicinity of the heated bottom plate along the central axis of the cylinder. The measurements were taken using a small microthermistor with a size of 125 μm approximately 30 times smaller compared with the typical thickness of the thermal boundary layer. The geometry of the sensor has been optimized to prevent that the connecting wires cross the temperature gradient. This provides a higher accuracy of the measured temperature particularly within the diffusive sublayer. The thermistor can be moved in steps of 10 μm perpendicular to the wall. In extension of our previous work we have complemented the temperature profile measurements by an independent heat flux sensor stuck at the surface of the heating plate. The sensor of a diameter of 20 mm and a thickness of 1 mm was mounted $0.01D$ away from the central axis. It measures the heat flux density q_w at the wall with an accuracy of better than 5% and permits us to calculate the temperature gradient at the surface using

$$\frac{dT}{dz} = -\frac{q_w}{\lambda(T)}, \tag{1}$$

with $\lambda(T)$ being the heat conductivity of the fluid. That additional piece of information allows us to validate the temperature measurements particularly very close to the wall and to accurately determine the thickness of the diffusive sublayer.

III. RESULTS

We start the discussion of our results which are summarized in Table I with the profiles of the normalized mean temperature $\Theta(z)$:

$$\Theta(z) = [T_h - T(z)]/(T_h - T_b). \tag{2}$$

Here T_h is the temperature of the heating plate and T_b is the temperature of the well-mixed core of the cell. We have measured profiles at a fixed aspect ratio $\Gamma=1.13$ and various Rayleigh numbers between $Ra=5.20\times10^{10}$ and $Ra=9.59\times10^{11}$ which is significantly below the critical Rayleigh number at which the boundary layer is expected to become turbulent [8]. We fixed the average of the heating and the cooling plate temperature $(T_h+T_c)/2$ at 30 °C except for the highest Ra number of $Ra=9.59\times10^{11}$. The wall-normal distance z is scaled by the boundary layer thickness δ_{th} from the locally measured heat flux at the wall with

$$\delta_{th} = \frac{\lambda(T_h - T_b)}{q_w}. \tag{3}$$

In Fig. 1 we plot two typical examples of $\Theta(z/\delta_{th})$ at two different Ra, namely, $Ra=1.60\times10^{11}$ and $Ra=8.32\times10^{11}$ (the latter might be slightly beyond the Boussinesq approximation). The dotted line indicates the nondimensional temperature gradient $d\Theta/d(z/\delta_{th})$ at the wall obtained according to Eq. (1). In close vicinity to the surface the measured temperature profile is nearly a linear function of z whose slope agrees well with the gradient computed from the independently measured wall heat flux. However, this region which we identify as the diffusive sublayer is very small and covers only a fluid layer of $z/\delta_{th}<0.05$. The question is: why is the linear range of the measured temperature profile that small and might it be an effect of the temperature dependency of the thermal conductivity of air?

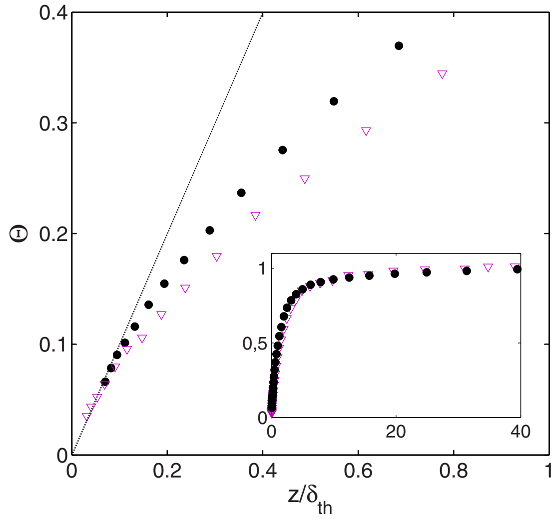

FIG. 1. (Color online) Profiles of the normalized mean temperature very close to the surface of the heating plate at $\mathrm{Ra}=1.60\times10^{11}$ (▽) and $\mathrm{Ra}=8.32\times10^{11}$ (●) and the corresponding temperature gradient $d\Theta/d(z/\delta_{th})$ obtained from the local heat flux sensor. The wall distance z is scaled by the thickness of the thermal boundary layer $\delta_{th}=H/2\mathrm{Nu}_l$. The inset shows the entire profiles.

According to Fourier's law of heat conduction the diffusive heat flux $q_d=\lambda[T(z)]dT/dz$ must be a constant as long as heat is exclusively transported by diffusion. For an intermediate Rayleigh number of $\mathrm{Ra}=5.42\times10^{11}$ which is representative for the full range of Ra investigated, we plot q_d/q_w over z/δ_{th} in Fig. 2. The behavior of the conductive heat flux clearly shows that the convection sets in closer to the surface of the plate than commonly assumed. According to the traditional view one would have expected that this curve stays close to 1 in a predominant domain of the thermal boundary layer. Our result shows that this is not the case. For $\mathrm{Ra}=5.42\times10^{11}$ a share of 10% of the entire heat transfer is convective at a position $z/\delta_{th}\approx0.045$. It corresponds to an absolute distance of $z\approx0.1$ mm in our large-scale experimental facility. This is remarkable since it means that (i) thermal plumes apparently penetrate very deep into the boundary layer and disturb the laminar structure of the flow and (ii) advection inside the boundary layer starts at significantly lower Rayleigh numbers than predicted in Ref. [8]. At a distance $z/\delta_{th}\approx0.25$ convection and diffusion equals at this specific Rayleigh number. We wish to mention here that this insight into the fine structure of the very inner fraction of the thermal boundary layer has been only feasible by a redesigned temperature sensor and the independent local heat flux measurement which ensures that particularly the near-wall measurements are accurate. However, it must be confessed as well that temperature measurements inside the diffusive sublayer require small sensors of the order of few microns which are commercially not available at the moment.

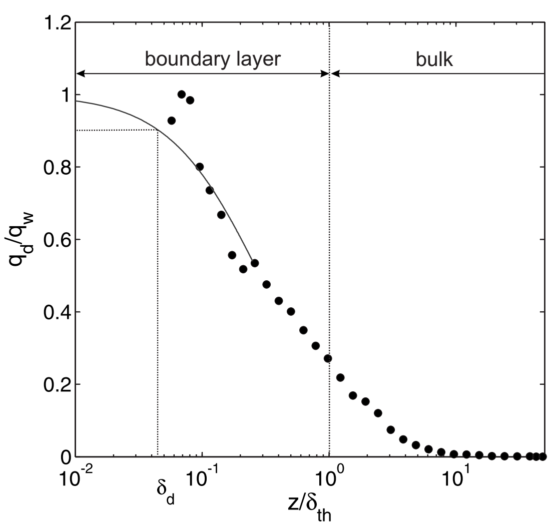

FIG. 2. Profile of the normalized diffusive heat flux q_d/q_w throughout the heating plate boundary layer at $\mathrm{Ra}=5.42\times10^{11}$. The distance z is scaled by the thickness of the thermal boundary layer δ_{th}. The solid line is a regression fit and δ_d is the thickness of the diffusive sublayer at which $q_d=0.9q_w$.

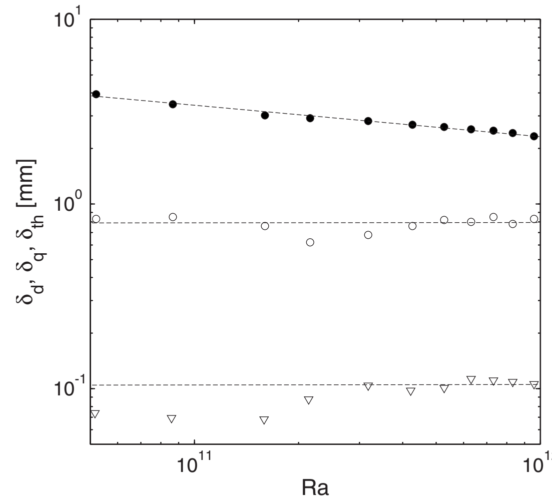

FIG. 3. Thickness of the diffusive sublayer δ_d (▽), thickness of the fluid layer δ_q in which $q_d\geq q_c$ (○) compared with the local boundary layer thickness $\delta_{th}=H/2\mathrm{Nu}_l$ (●) over Ra.

The next question naturally arising here is: how do the typical boundary layer length scales depend on the Rayleigh number? In order to determine the thickness of the diffusive sublayer δ_d we fit the profile of the diffusive heat flux $q_d(z/\delta_{th})$ in the interval $0<z/\delta_{th}<0.25$ by a function fulfilling the conditions $q_d|_{z/\delta=0}=q_w$ and $dq_d/d(z/\delta_{th})|_{z/\delta=0}=0$. We define δ_d as the position at which $q_d=0.9q_w$. Another point of particular interest is the position δ_q at which diffusive and convective heat flux q_d and q_c equals. We plot both quantities together with the local boundary layer thickness δ_{th} in Fig. 3. For the range of Ra investigated here, both δ_d and δ_q are significantly smaller than the thickness of the thermal boundary layer δ_{th}. While the first two quantities do not vary, the local boundary layer thickness δ_{th} obtained from a direct heat flux measurement at the wall decreases with increasing Ra according to a power law $\delta_{th}=C\times(\mathrm{Ra}/\mathrm{Ra}^*)^{\alpha}$ with $C=3.816\pm0.119$ m, $\alpha=-0.169\pm0.017$, and $R^*=5.2\times10^{10}$. In this formula R^* has been introduced as a reference to reduce the statistical uncertainty of the prefactor C. Compared with results from previous work (see, e.g., Lui and Xia, $\alpha=-0.285$ [14] or du Puits *et al.*, $\alpha=-0.254$ [17]) the

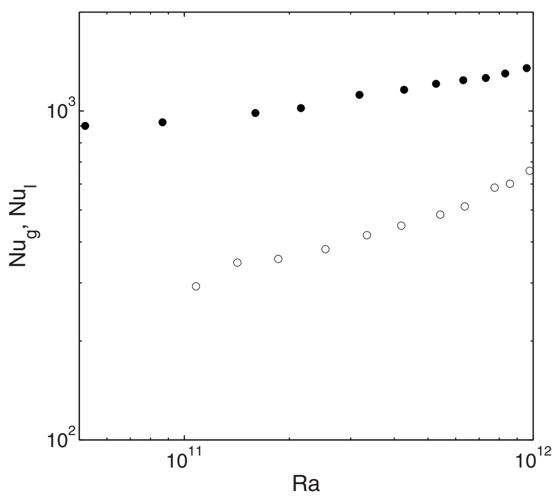

FIG. 4. Local Nusselt number Nu_l (●) [see Eq. (4)] as a function of the Rayleigh number for constant aspect ratio $\Gamma=1.13$ and global Nusselt number Nu_g (○) from previous measurements [17].

exponent α is smaller than those exponents reported there. However, one has to consider that we derived the thermal boundary layer thickness from a direct heat flux measurement at the wall which is definitely more accurate than those values obtained from the near-wall slope of the mean temperature profile $T(z)$.

Finally let us discuss the dependence of the local heat flux on the Rayleigh number for the purpose of comparison with the global average throughout the RB cell. Our direct heat flux measurement at the center of the heating plate permits us computing a local Nusselt number which we define as

$$\mathrm{Nu}_l = \frac{q_w H}{2\lambda_h (T_h - T_b)}. \tag{4}$$

This quantity is plotted in Fig. 4 together with the global Nusselt number Nu_g from our previous measurements [17]. It is remarkable that in cylindrical RB cells of aspect ratio close to unity the heat flux concentrates in the cell center with decreasing Rayleigh numbers. For the lowest value $Ra=5.20\times 10^{10}$ the local heat flux at the plate center exceeds the average by a factor of 3.9 while for the highest $Ra=9.59\times 10^{11}$ the ratio between both quantities amounts to a value of only 2.2. In harmony with the ideas of Niemela's and Sreenivasan's work [21] and the measurements of Xia *et al.* [22] the homogenization of the local heat flux over the surface of the plate might be caused by the change in the large convection roll from a rather elliptical shape with a relatively small boundary layer at the cell center to a more rectangular shape where the boundary layer thickness is more homogeneous. It is not ruled out that this mechanism causes the deviation of β in the scaling of the global heat transfer $Nu \sim Ra^{\beta}$ in highly turbulent RB convection in experiments with aspect ratio of the order of unity [23] while large aspect ratio cells without a distinct mean flow exhibit an exponent $\beta=1/3$ [24].

IV. CONCLUSION

The main result of our work is that the diffusive sublayer in highly turbulent convection is very small compared with the entire thermal boundary layer. For Rayleigh numbers $5.20\times 10^{10} < Ra < 9.59\times 10^{11}$ and a fixed aspect ratio of $\Gamma=1.13$ temperature measurements in the vicinity of the heated plate complemented by a direct heat flux measurement at its surface show a ratio between the thicknesses of the diffusive sublayer δ_d and the entire thermal boundary layer δ_{th} on the order of 1/25. While the latter clearly scales with the Rayleigh number the thickness of the diffusive sublayer remains approximately constant. A particular feature of confined turbulent convection in enclosures with aspect ratio of order unity is the strong concentration of the heat flux in the center of the plates. This effect decreases with rising Ra numbers.

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